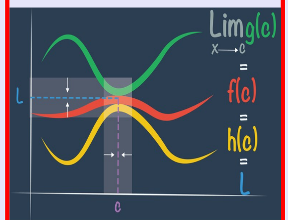
Math 261

Fall 2022

Lecture 4



Class QZ 1

Solve $4x^2 + 5x - 9 = 0$ using quadratic formula.

Express answers in a Solution Set.

$4x^2 + 5x - 9 = 0$

$ax^2 + bx + c = 0$

$a = 4, \quad b = 5, \quad c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(4)(-9)}}{2(4)}$$

$$= \frac{-5 \pm \sqrt{25 + 144}}{8} = \frac{-5 \pm \sqrt{169}}{8}$$

$$= \frac{-5 \pm 13}{8}$$

$$x = \frac{-5 + 13}{8} = \frac{8}{8} = 1$$

$$x = \frac{-5 - 13}{8} = \frac{-18}{8} = \frac{-9}{4}$$

$\Rightarrow \boxed{\text{Solution } \left\{ -\frac{9}{4}, 1 \right\}}$

How to evaluate limits:

1) Plug it in.

$$\lim_{x\to 2} (x^2 - 6x) = 2^2 - 6(2) = 4 - 12 = \boxed{-8}$$

$$\lim_{x\to -2} \frac{x^2+2x}{x+2} = \frac{(-2)^2+2(-2)}{-2+2} = \frac{4-4}{-2+2} = \frac{0}{0}$$

Indeterminate form

Try Simplifying

$$\lim_{x\to -2} \frac{x^2+2x}{x+2} = \lim_{x\to -2} \frac{x\cancel{(x+2)}}{\cancel{x+2}} = \lim_{x\to -2} x = \boxed{-2}$$

Find $\displaystyle\lim_{x\to -3} \frac{x^2+5x+6}{x^2-9} = \frac{(-3)^2+5(-3)+6}{(-3)^2-9} = \frac{0}{0}$

I.F.

$$\lim_{x\to -3} \frac{(x+2)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \lim_{x\to -3} \frac{x+2}{x-3} = \frac{-3+2}{-3-3} = \boxed{\frac{1}{6}}$$

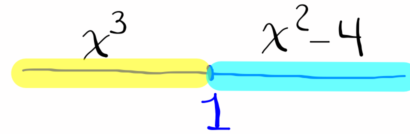
Evaluate $\displaystyle\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ I.F.

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{\underbrace{(\sqrt{x}-2)(\sqrt{x}+2)}_{(A-B)(A+B)}} = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{\underbrace{(\sqrt{x})^2-2^2}_{A^2-B^2}}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}} = \lim_{x\to 4} (\sqrt{x}+2)$$

$$= \sqrt{4}+2 = \boxed{4}$$

$$f(x)=\begin{cases} x^3 & \text{if } x<1 \\ x^2-4 & \text{if } x\geq 1 \end{cases}$$



1) $\lim\limits_{x\to 1^-} f(x)$

$=\lim\limits_{x\to 1^-} x^3 = 1^3 = \boxed{1}$

2) $\lim\limits_{x\to 1^+} f(x)$

$=\lim\limits_{x\to 1^+} (x^2-4) = 1^2-4 = \boxed{-3}$

3) $\lim\limits_{x\to 1} f(x)$ D.N.E.

one-sided limits are not equal.

4) $f(1) = 1^2-4 = \boxed{-3}$

Evaluate $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1} = \dfrac{\frac{1}{1}-1}{1-1} = \dfrac{0}{0}$ I.F.

LCD $= x$

$\lim\limits_{x\to 1} \dfrac{x\left(\frac{1}{x}-1\right)}{x(x-1)} = \lim\limits_{x\to 1} \dfrac{\overset{-1(\cancel{x-1})}{1-x}}{x(\cancel{x-1})}$

$= \lim\limits_{x\to 1} \dfrac{-1}{x} = \dfrac{-1}{1} = \boxed{-1}$

Evaluate

$\lim\limits_{x\to 2} \dfrac{x^2-4x+4}{x^2+x-6} = \dfrac{2^2-4(2)+4}{2^2+2-6} = \dfrac{0}{0}$ I.F.

$\lim\limits_{x\to 2} \dfrac{x^2-4x+4}{x^2+x-6} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x+3)} = \lim\limits_{x\to 2} \dfrac{x-2}{x+3} = \dfrac{2-2}{2+3} = \dfrac{0}{5}$

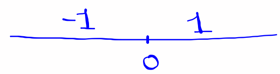
$= \boxed{0}$

Evaluate $\lim\limits_{x \to 0} \dfrac{x}{|x|} = \dfrac{0}{0}$ I.F.

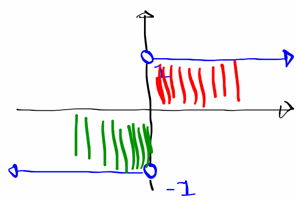
$f(x) = \dfrac{x}{|x|} \quad x \neq 0$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x}{-x} = -1 & \text{if } x < 0 \\ \dfrac{x}{x} = 1 & \text{if } x > 0 \end{cases}$$



$\lim\limits_{x \to 0^-} \dfrac{x}{|x|} = -1$



$\lim\limits_{x \to 0^+} \dfrac{x}{|x|} = 1$

$\lim\limits_{x \to 0} \dfrac{x}{|x|}$ D.N.E.

$\dfrac{-1}{|-1|} = \dfrac{-1}{1} = -1 \qquad \dfrac{5}{|5|} = \dfrac{5}{5} = 1$

$\dfrac{-4}{|-4|} = \dfrac{-4}{4} = -1 \qquad \dfrac{-20}{|-20|} = -1$

$f(x)$ is continuous at $x = a$ if

$$\lim_{x \to a} f(x) = f(a)$$

No gap
No jump
No hole

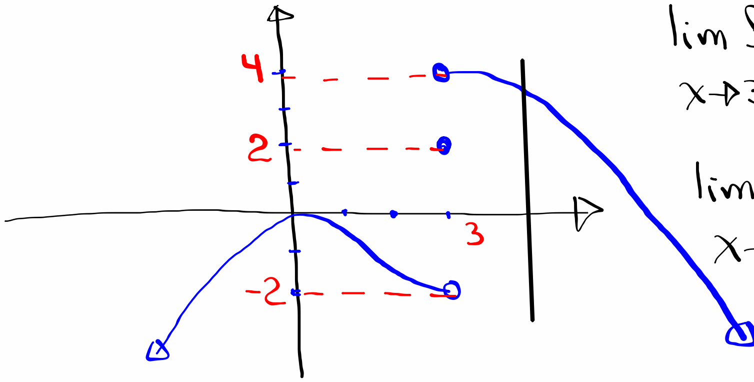
ex: Is $f(x) = x^2 + 3x$ cont. at $x = 2$?

$f(2) = 2^2 + 3(2) = 10$

$\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} (x^2 + 3x) = 2^2 + 3(2) = 10$

Since $\lim\limits_{x \to 2} f(x) = f(2)$, then $f(x)$ is Continuous at $x = 2$.

Consider the graph of $f(x)$ given below



$\lim_{x \to 3^-} f(x) = -2$

$\lim_{x \to 3^+} f(x) = 4$

$\lim_{x \to 3} f(x)$ D.N.E.

$f(3) = 2$

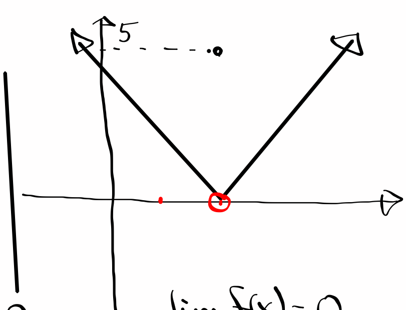
$f(x)$ is not cont. at $x = 3$

Consider the piece-wise function below

Graph

Recall $y = |x|$ what about $|x-2|$?

$$f(x) = \begin{cases} |x-2| & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$



$\lim_{x \to 2^+} f(x) = 0$

$\lim_{x \to 2^-} f(x) = 0$

$\lim_{x \to 2} f(x) = 0$

Is $f(x)$ cont. at $x = 2$? NO

$\lim_{x \to 2} f(x) \neq f(2)$

$0 \neq 5$

Find the difference quotient for any quadratic function. Evaluate final ans. for $h=0$.

$f(x)=ax^2+bx+c$
$a\neq 0$

$$\frac{f(x+h)-f(x)}{h}=\frac{a(x+h)^2+b(x+h)+c-ax^2-bx-c}{h}$$

$$=\frac{\cancel{ax^2}+2axh+ah^2\cancel{+bx}+bh\cancel{+c}\cancel{-ax^2}\cancel{-bx}\cancel{-c}}{h}$$

$$=\frac{2axh+ah^2+bh}{h}=\frac{\cancel{h}(2ax+ah+b)}{\cancel{h}}$$

$=2ax+ah+b$

Evaluate for $h=0$

$2ax+b$